Pós-Graduação em Matemática
Universidade Federal Fluminense

Ciclo Anual de Palestras, Seminários e Minicursos do Programa de Pós-Graduação em Matemática da UFF

Título:

A Taste of Lefschetz Properties: from Togliatti Systems to Osculating Defects

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Data: 18/11
Hora: 15h
Local: Sala 407 - Bloco H - Campus Gragoatá

Resumo

The Weak and Strong Lefschetz Properties (WLP/SLP) of standard graded Artinian algebras admit a geometric interpretation in characteristic zero. For an artinian ideal $I \subset\mathbb{K}[x_0,\dots,x_n]$ generated by at most $\binom{n+d-1}{n-1}$ forms of degree d, the failure of the WLP in degree $d-1$ is equivalent to the projected Veronese variety defined by $|I_d|$ satisfying a Laplace equation of order $d-1$. Such ideals are called Togliatti systems. We show that the failure of the SLP for any standard graded Artinian K-algebra, in any degree and any range, is equivalent to the osculating defect of a certain rational variety, with no assumption that the defining ideal is equigenerated.

As a consequence, the vanishing of a higher Hessian acquires a geometric interpretation, extending the Gordan–Noether criterion. In the second part of the talk we focus on monomial Togliatti systems. In this case, the variety is toric, and the Togliatti condition translates into a statement regarding lattice points within the dilated simplex $d\Delta_n$. We provide an overview of the classification of minimal systems, discussing the possible number of generators, the hypersurface associated with each system, and various open problems.