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Exponential Series Defined by Sequences and Their Connection with the Riemann Zeta Function

Abstract: We study exponential series associated with real sequences,

$$\chi_{c_n}(s) = \sum_{n=1}^{\infty} e^{-sc_n},$$

where $(c_n)_{n\in\mathbb{N}}$ is a sequence of real numbers and $s \in \mathbb{C}$.

We establish convergence criteria for these series in terms of the asymptotic parameter

$$L = \lim_{n\to\infty} \frac{\ln n}{c_n},$$

showing that L determines the convergence threshold of $\chi_{c_n}(s)$.

When $0 < L < \infty$, we prove that the corresponding exponential series is asymptotically governed by the scaled Riemann zeta function

$$\zeta_L(s) = \zeta\left(\frac{s}{L}\right).$$

More precisely, after modifying finitely many terms if necessary, $\chi_{c_n}(s)$ can be sandwiched between arbitrarily close scaled zeta functions and can approximate $\zeta_L(s)$ arbitrarily well in the region $\Re(s) > L$.

We derive alternative integral representations for scaled zeta functions and study special families of exponential series, including the case $c_n = n^p$.