Hamilton-Jacobi-Bellman Equation for a Riemann-Stieltjes Control Problem

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In this work, we deal with the existence of optimal trajectories of a Riemann-Stieltjes control problem, concretely, we consider the following infinite dimensional problem $(P)_{s,\varphi}$ given by

$$\min \int_{\Omega} g(x(T,\omega),\omega)d\mu(\omega),$$

s.t.

$$\begin{cases} \dot{x}(t,\omega) = f(t,x(t,\omega),u(t),\omega), & \text{a.e. } t \in [s,T], \\ x(s,\omega) = \varphi(\omega), \\ u(t) \in U(t), \quad \text{a.e. } t \in [s,T], & \text{whit } U : [0,T] \rightsquigarrow \mathbb{R}^m \quad \text{a multifunction,} \end{cases} \tag{1}$$

the dynamics is measurable in time and (Ω, d_Ω, μ) is a compact metric measure space, with control $u \in L^\infty(0,T;U)$ and initial data $\varphi \in L^2(\mu,\Omega;\mathbb{R}^n)$ at time $s \in [0,T]$. First, we prove that under some hypothesis on the dynamics and the measure μ, the set of trajectories $S_{[s,T]}(\varphi)$ of (1) is compact in $C([s,T];L^2(\mu,\Omega;\mathbb{R}^n))$, then we give a characterization of the lower semicontinuity of the associated value function

$$V(s,\varphi) = \inf\left\{\int_{\Omega} g(x(T,\omega),\omega)d\mu(\omega) : x \in S_{[s,T]}(\varphi)\right\}$$

which leaves with the result of the existence of optimal trajectories.

The existence of minimizers joint with invariance principles and the *Dynamic Programming Principle* will open the way to prove that the value function defined above is the unique lower semicontinuous solution of the Hamilton-Jacobi-Bellman equation defined in an infinite-dimensional space.

Bibliography

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