THE GEOMETRY OF NUMBERS, FROM A FOURIER-ANALYTIC PERSPECTIVE

SINAI ROBINS

ABSTRACT. We extend the Bombieri-Siegel formula from the geometry of numbers, by studying a lattice sum of the cross-covariogram of two bounded sets $A, B \subset \mathbb{R}^d$. Using a variation of Poisson summation, our extension also refines the summation index of the Bombieri-Siegel formula, allowing us to obtain some interesting applications.

One of the consequences of these results is a new characterization of multi-tiling Euclidean space by translations of a compact set. Another consequence is a spectral formula for the volume of a compact set. Finally, we give an application to arithmetic combinatorics, namely an identity for finite sums of discrete covariograms over any set of integer points in $\mathbb{Z}^d$. As a consequence, we arrive at an equivalent condition for multi-tiling $\mathbb{Z}^d$ by any finite set of integer points.

This is joint work with my Ph.D. student Michel Faleiros Martins.